Error surface is rugged …

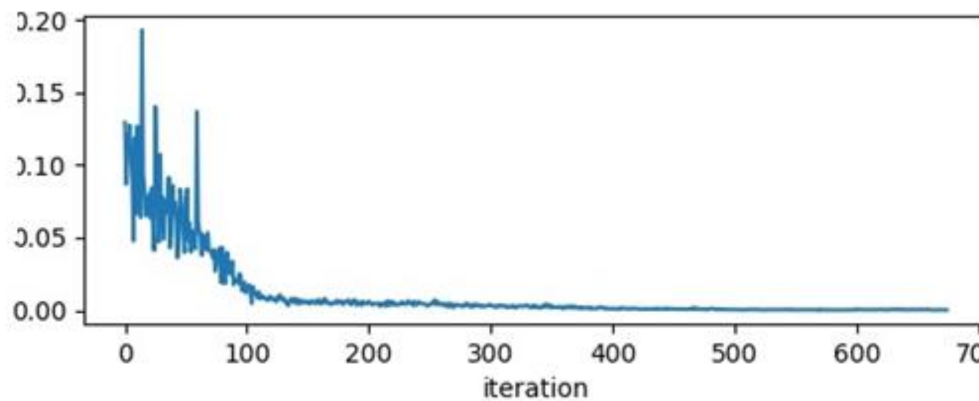
Tips for training: **Adaptive Learning Rate**

Also refer to: http://www.deeplearningbook.org/contents/optimization.html

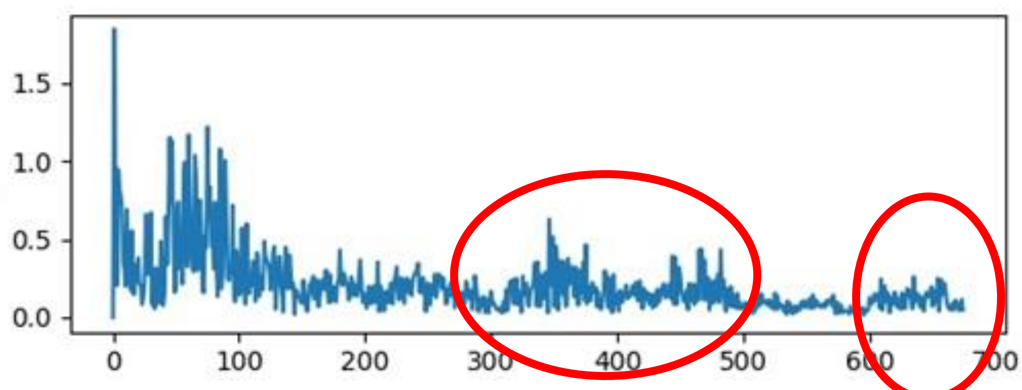
Training stuck ≠ Small Gradient

- People believe training stuck because the parameters are around a critical point …

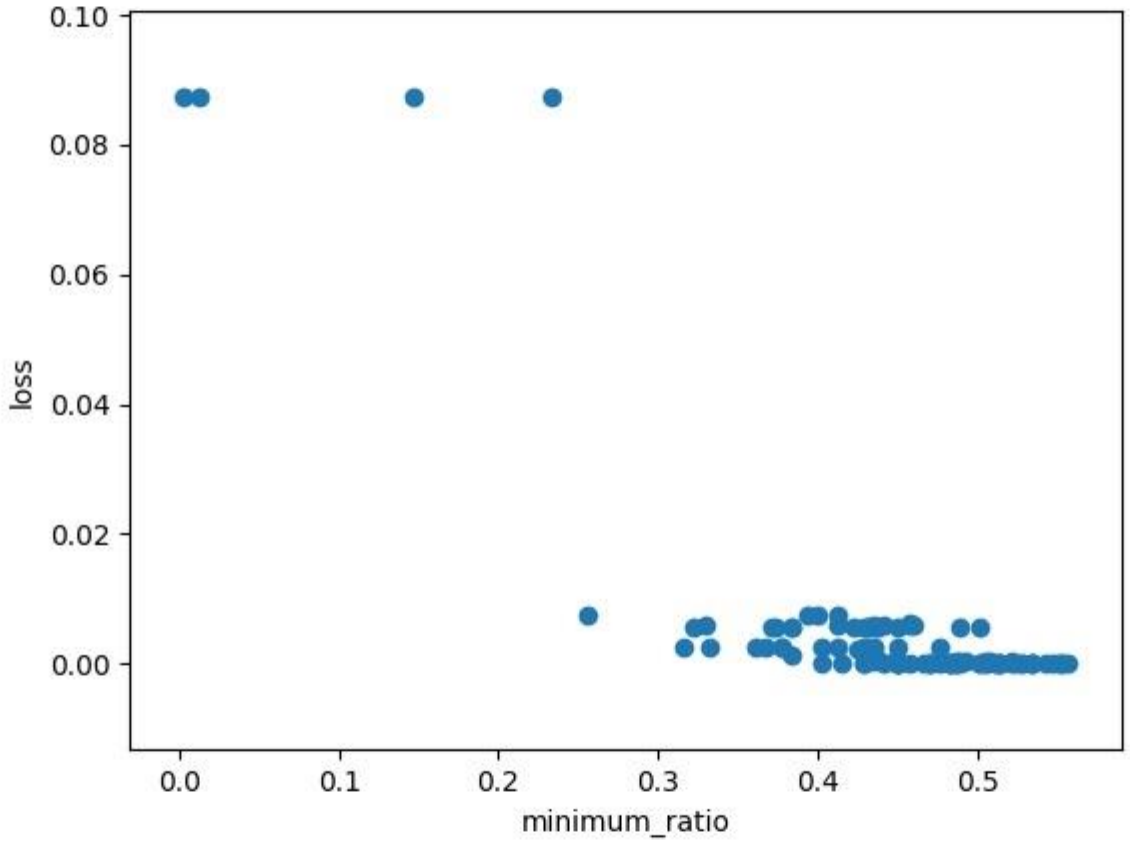
<u>MNIST</u> loss



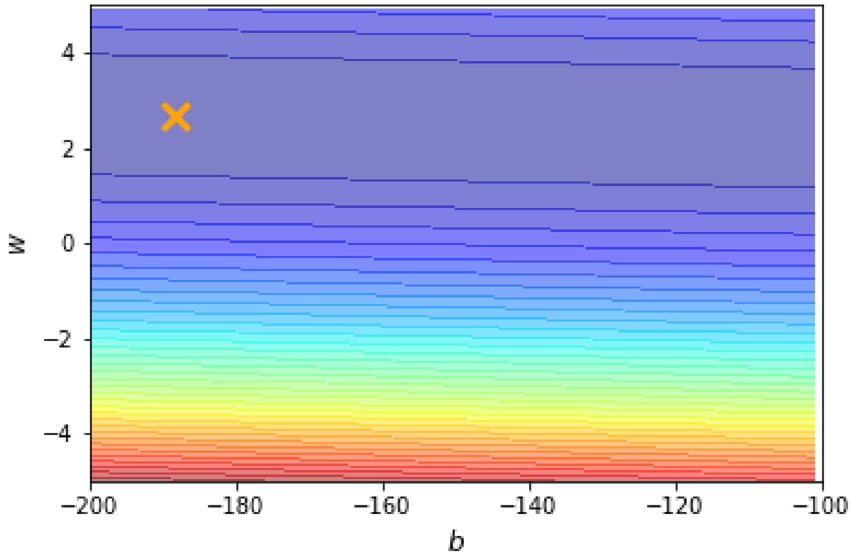
norm of gradient



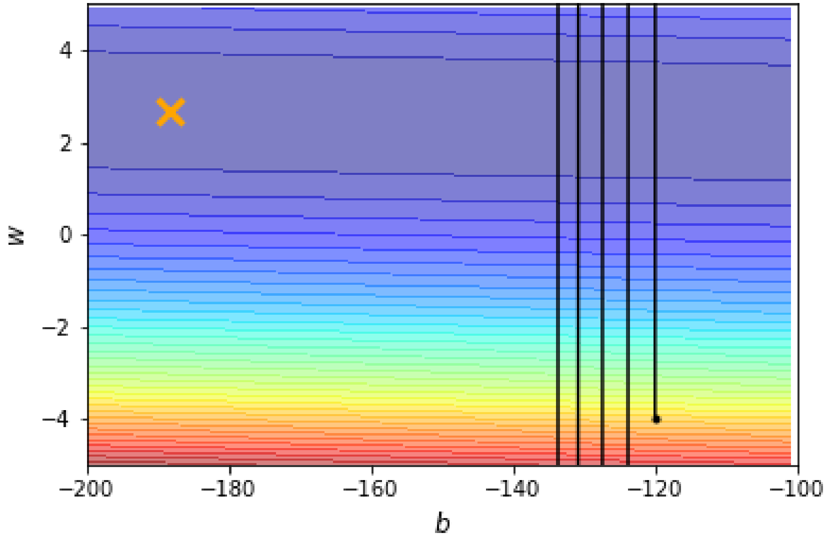
Wait a minute …



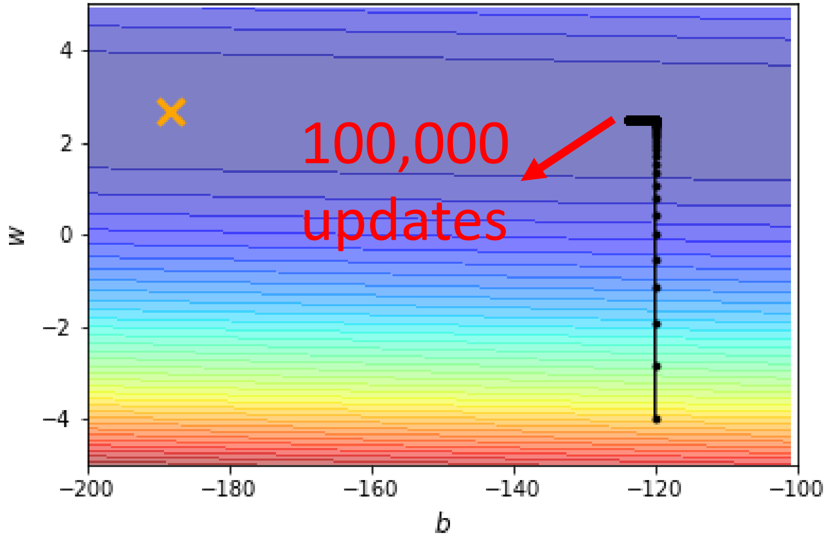
Training stuck without critical points



Learning rate **cannot** be **one-size-fits-all**

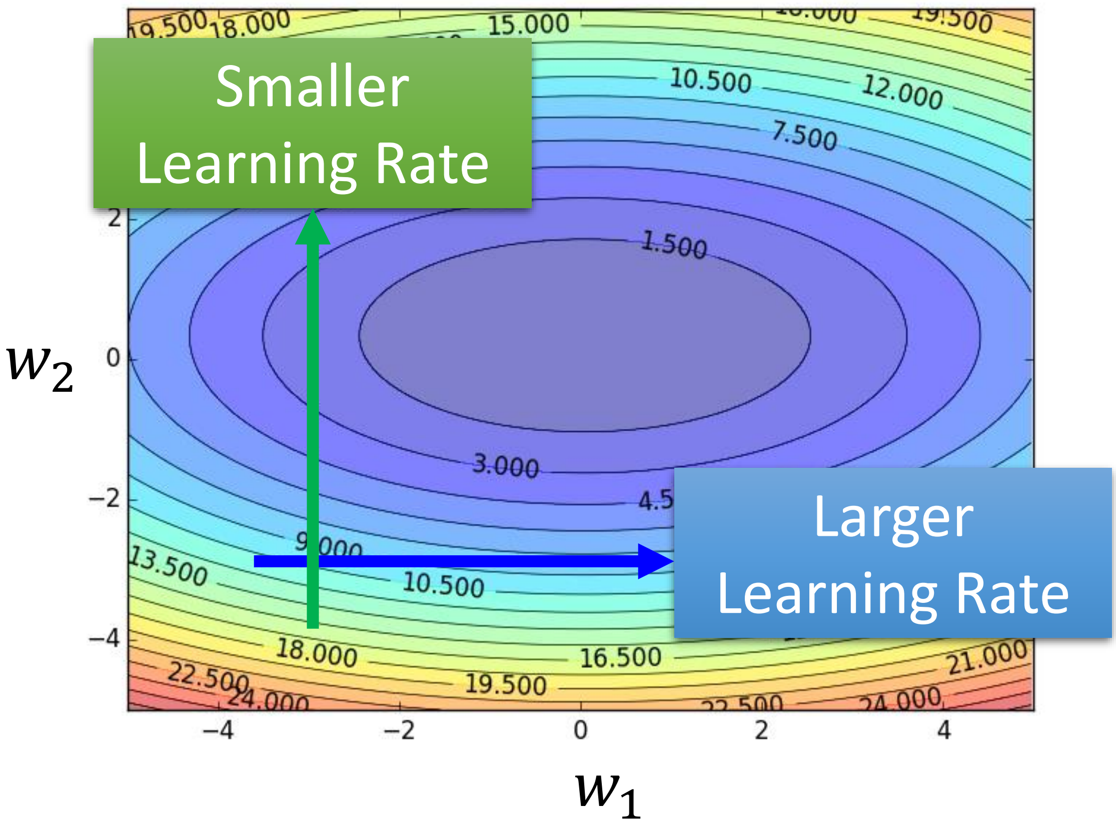


$\eta = 10^{-2}$



$\eta = 10^{-7}$

Different parameters needs different learning rate



Update one parameter:

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \boxed{\eta}\,\boldsymbol{g}_i^{t}$$

$$\boldsymbol{g}_i^{t} = \frac{\partial L}{\partial \boldsymbol{\theta}_i}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^t}$$

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \boxed{\frac{\eta}{\sigma_i^t}}\,\boldsymbol{g}_i^{t}$$

Parameter dependent

Root Mean Square

$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^{\boldsymbol{t}}$$

$$\boldsymbol{\theta}_i^{\mathbf{1}} \leftarrow \boldsymbol{\theta}_i^0 - \frac{\eta}{\sigma_i^0} \boldsymbol{g}_i^{\mathbf{0}} \qquad \sigma_i^0 = \sqrt{\left(\boldsymbol{g}_i^{\mathbf{0}}\right)^2}$$

$$\boldsymbol{\theta}_i^{\mathbf{2}} \leftarrow \boldsymbol{\theta}_i^1 - \frac{\eta}{\sigma_i^1} \boldsymbol{g}_i^{\mathbf{1}} \qquad \sigma_i^1 = \sqrt{\frac{1}{2}\left[\left(\boldsymbol{g}_i^{\mathbf{0}}\right)^2 + \left(\boldsymbol{g}_i^{\mathbf{1}}\right)^2\right]}$$

$$\boldsymbol{\theta}_i^3 \leftarrow \boldsymbol{\theta}_i^2 - \frac{\eta}{\sigma_i^2} \boldsymbol{g}_i^{\mathbf{2}} \qquad \sigma_i^2 = \sqrt{\frac{1}{3}\left[\left(\boldsymbol{g}_i^{\mathbf{0}}\right)^2 + \left(\boldsymbol{g}_i^{\mathbf{1}}\right)^2 + \left(\boldsymbol{g}_i^{\mathbf{2}}\right)^2\right]}$$

$$\vdots$$

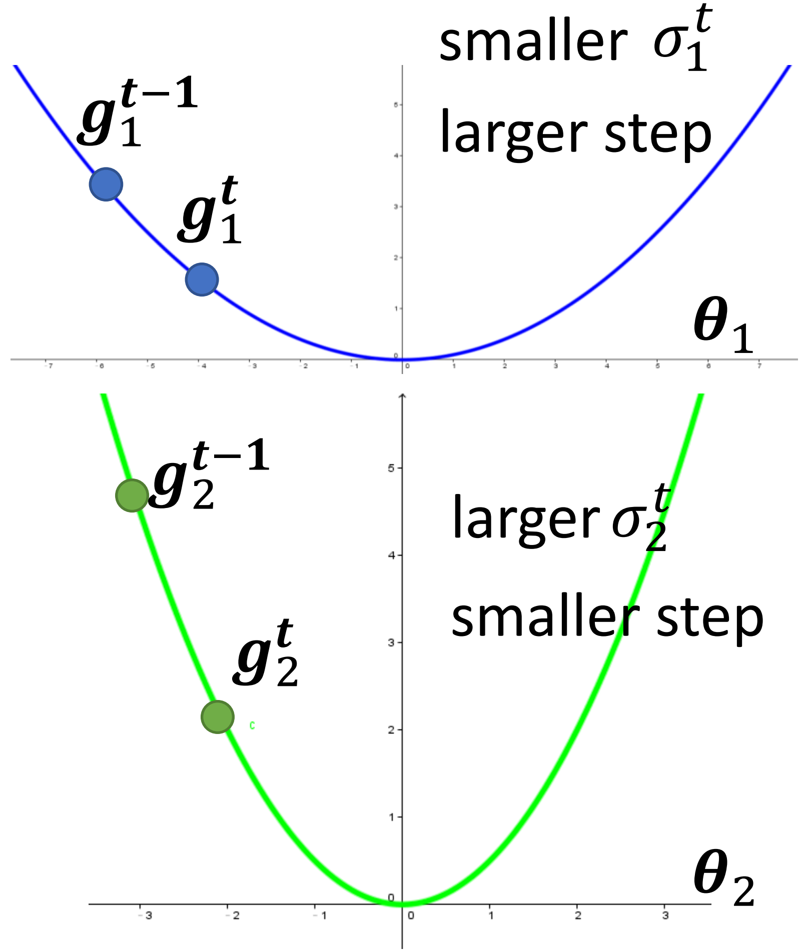
$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \frac{\eta}{\sigma_i^t} \boldsymbol{g}_i^{\boldsymbol{t}} \qquad \sigma_i^t = \sqrt{\frac{1}{t+1}\sum_{i=0}^{t}\left(\boldsymbol{g}_i^{\boldsymbol{t}}\right)^2}$$

Root Mean Square

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^t$$

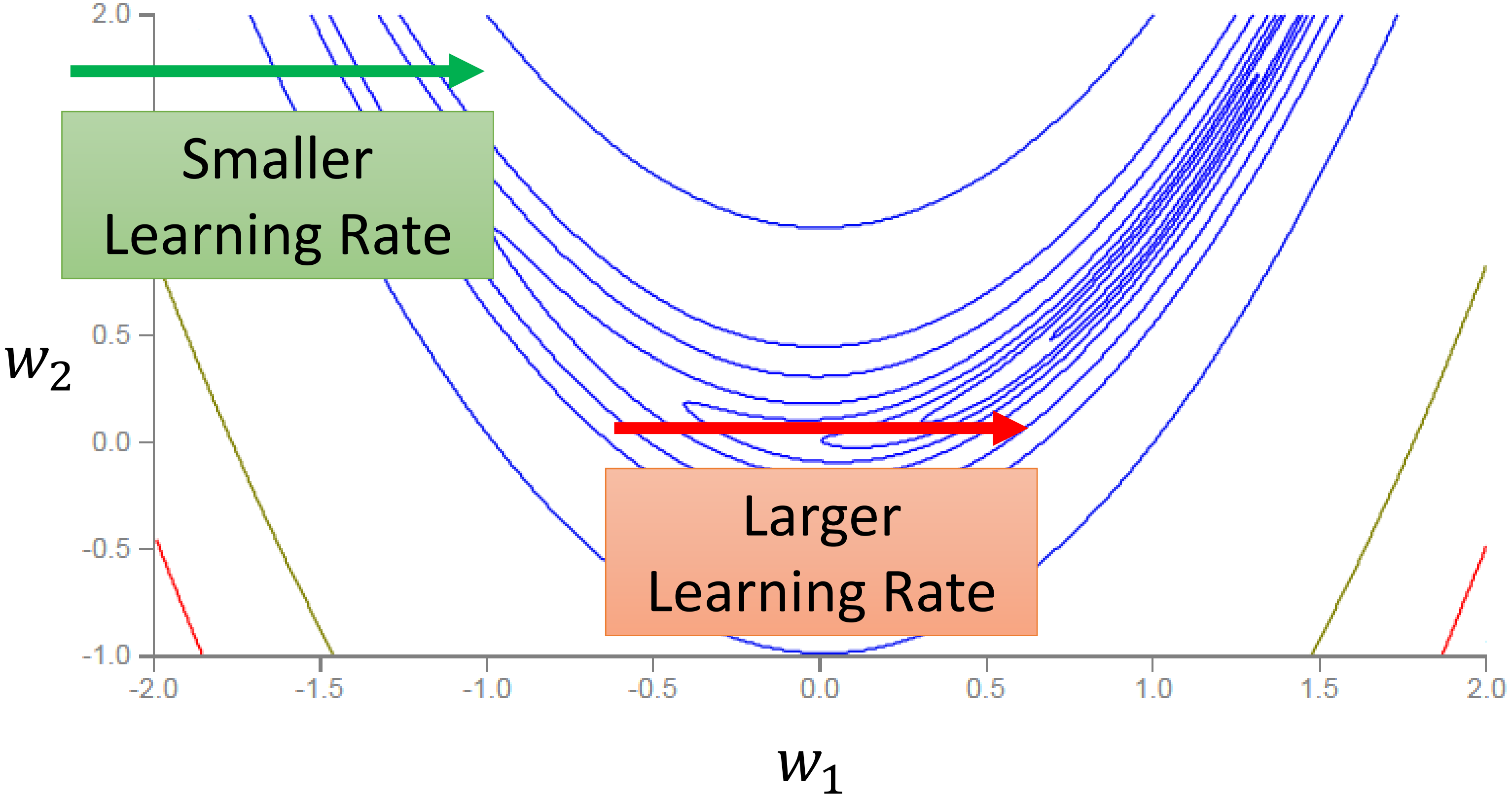
$$\sigma_i^t = \sqrt{\frac{1}{t+1}\sum_{i=0}^{t}\left(\boldsymbol{g}_i^t\right)^2}$$

Used in **Adagrad**



Learning rate adapts dynamically

Error Surface can be very complex.



RMSProp

$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^{\boldsymbol{t}}$$

$$\boldsymbol{\theta}_i^{\boldsymbol{1}} \leftarrow \boldsymbol{\theta}_i^0 - \frac{\eta}{\sigma_i^0} \boldsymbol{g}_i^{\boldsymbol{0}} \qquad \sigma_i^0 = \sqrt{\left(\boldsymbol{g}_i^{\boldsymbol{0}}\right)^2}$$

$$0 < \alpha < 1$$

$$\boldsymbol{\theta}_i^{\boldsymbol{2}} \leftarrow \boldsymbol{\theta}_i^1 - \frac{\eta}{\sigma_i^1} \boldsymbol{g}_i^{\boldsymbol{1}} \qquad \sigma_i^1 = \sqrt{\alpha\left(\sigma_i^0\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^{\boldsymbol{1}}\right)^2}$$

$$\boldsymbol{\theta}_i^3 \leftarrow \boldsymbol{\theta}_i^2 - \frac{\eta}{\sigma_i^2} \boldsymbol{g}_i^{\boldsymbol{2}} \qquad \sigma_i^2 = \sqrt{\alpha\left(\sigma_i^1\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^{\boldsymbol{2}}\right)^2}$$

$$\vdots$$

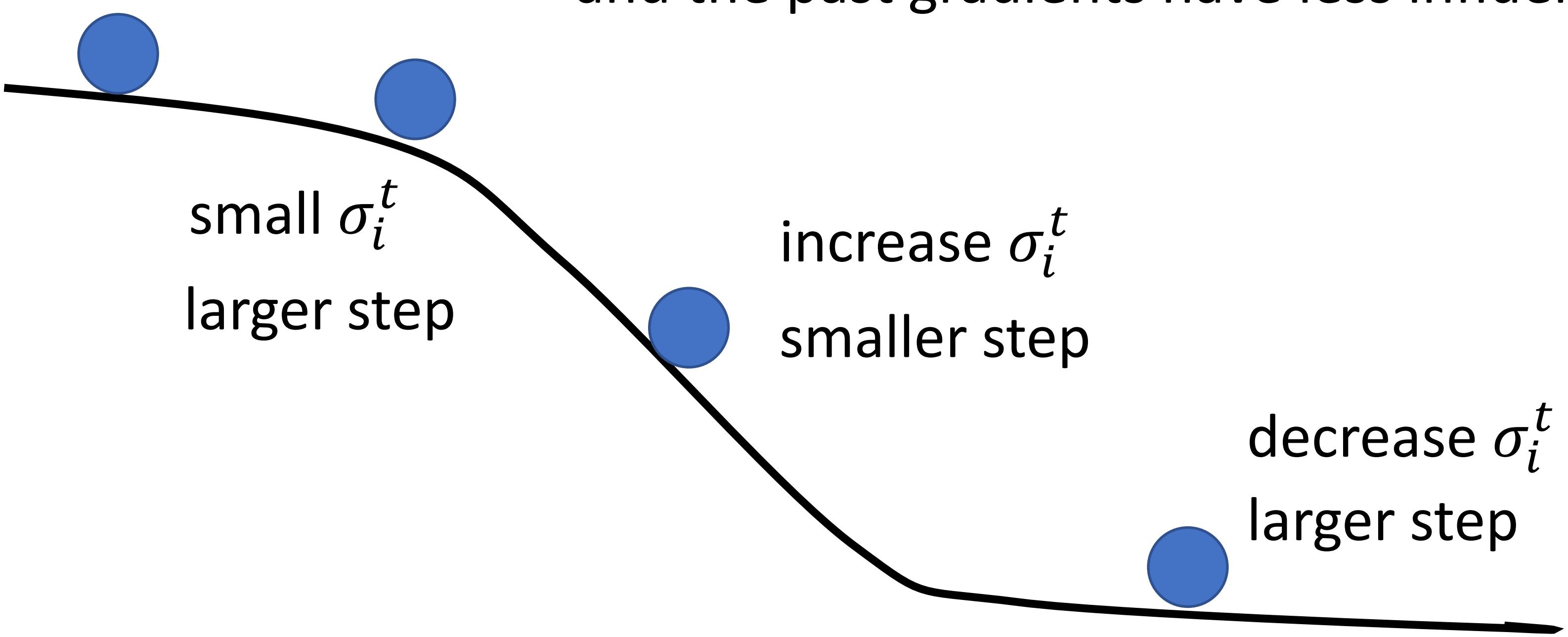
$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \frac{\eta}{\sigma_i^t} \boldsymbol{g}_i^{\boldsymbol{t}} \qquad \sigma_i^t = \sqrt{\alpha\left(\sigma_i^{t-1}\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^{\boldsymbol{t}}\right)^2}$$

RMSProp

$$0 < \alpha < 1$$

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^{t} \qquad \sigma_i^t = \sqrt{\alpha\left(\sigma_i^{t-1}\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^{t}\right)^2}$$

The recent gradient has larger influence,
and the past gradients have less influence.



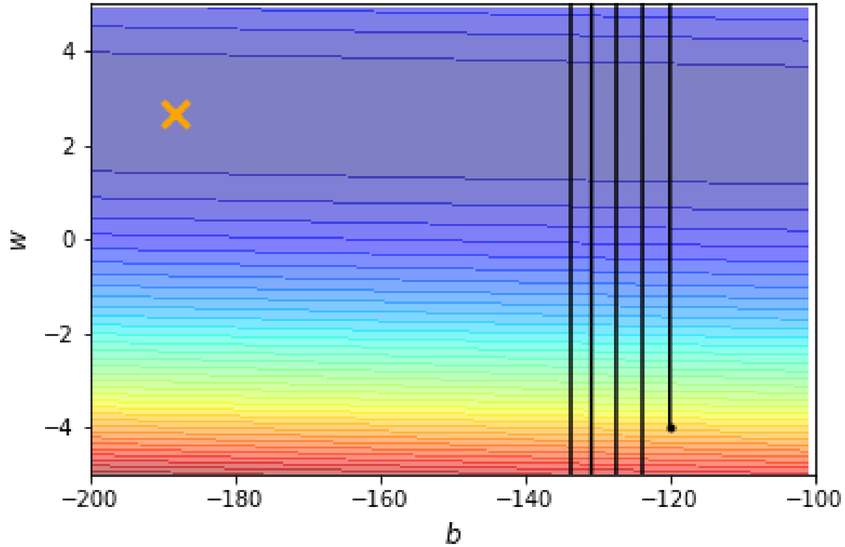
Original paper: https://arxiv.org/pdf/1412.6980.pdf

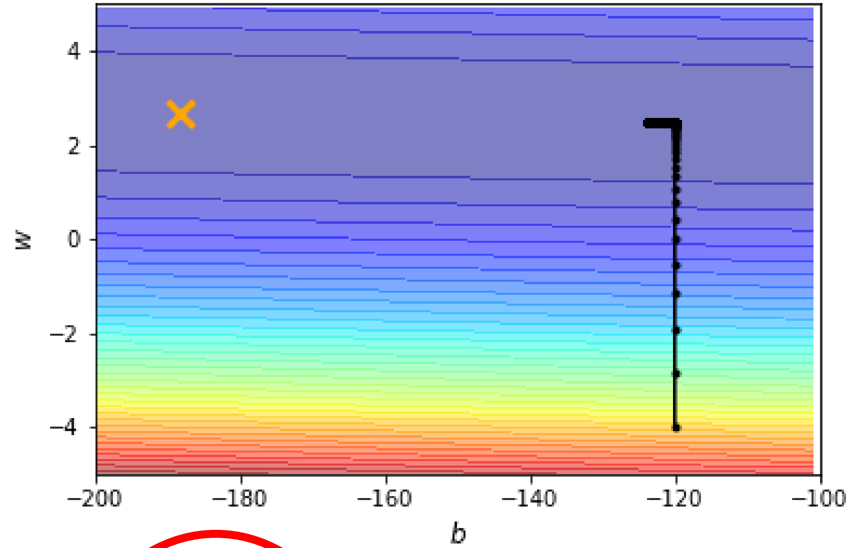
Adam: RMSProp + Momentum

Algorithm 1: *Adam*, our proposed algorithm for stochastic optimization. See section 2 for details, and for a slightly more efficient (but less clear) order of computation. g_t^2 indicates the elementwise square $g_t \odot g_t$. Good default settings for the tested machine learning problems are $\alpha = 0.001$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$. All operations on vectors are element-wise. With β_1^t and β_2^t we denote β_1 and β_2 to the power t.

Require: α: Stepsize
Require: $\beta_1, \beta_2 \in [0, 1)$: Exponential decay rates for the moment estimates
Require: $f(\theta)$: Stochastic objective function with parameters θ
Require: θ_0: Initial parameter vector
  $m_0 \leftarrow 0$ (Initialize 1st moment vector) → for momentum
  $v_0 \leftarrow 0$ (Initialize 2nd moment vector) → for RMSprop
  $t \leftarrow 0$ (Initialize timestep)
  while θ_t not converged **do**
    $t \leftarrow t + 1$
    $g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$ (Get gradients w.r.t. stochastic objective at timestep t)
    $m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$ (Update biased first moment estimate)
    $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$ (Update biased second raw moment estimate)
    $\widehat{m}_t \leftarrow m_t / (1 - \beta_1^t)$ (Compute bias-corrected first moment estimate)
    $\widehat{v}_t \leftarrow v_t / (1 - \beta_2^t)$ (Compute bias-corrected second raw moment estimate)
    $\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \widehat{m}_t / (\sqrt{\widehat{v}_t} + \epsilon)$ (Update parameters)
  end while
  return θ_t (Resulting parameters)

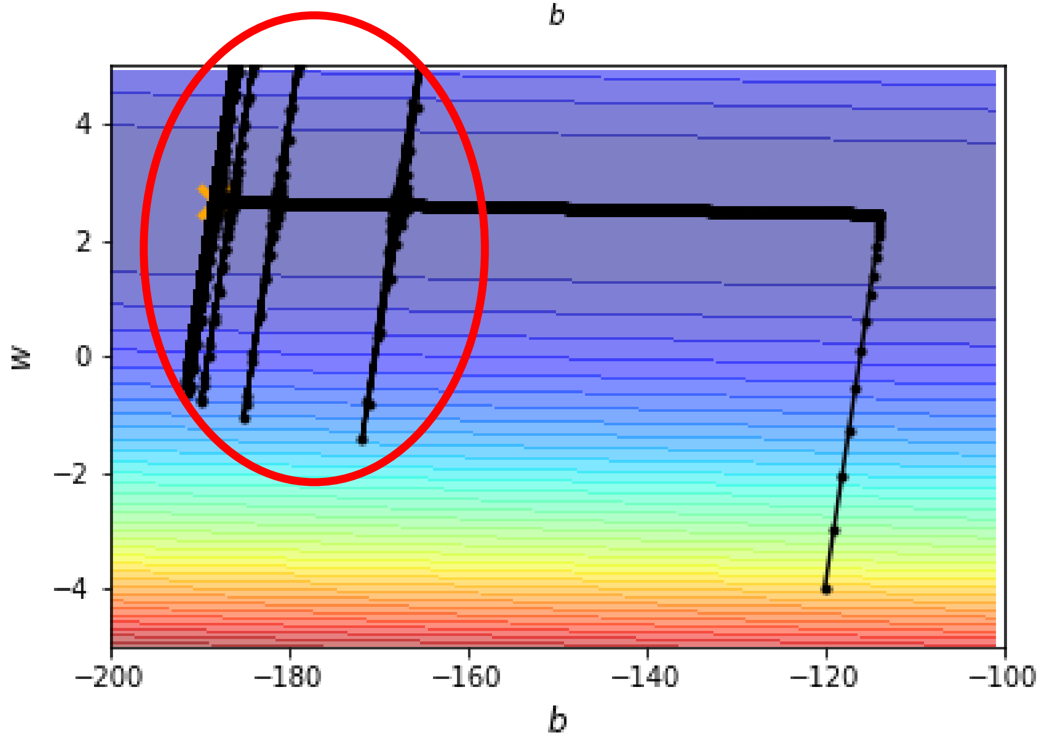
Without Adaptive Learning Rate





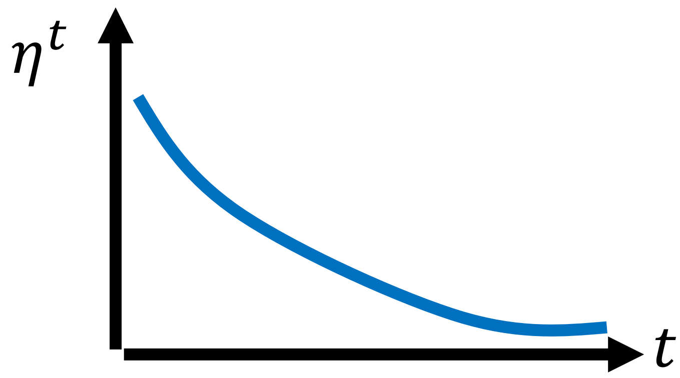
$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^t$$

$$\sigma_i^t = \sqrt{\frac{1}{t+1} \sum_{i=0}^{t} \left(\boldsymbol{g}_i^t\right)^2}$$



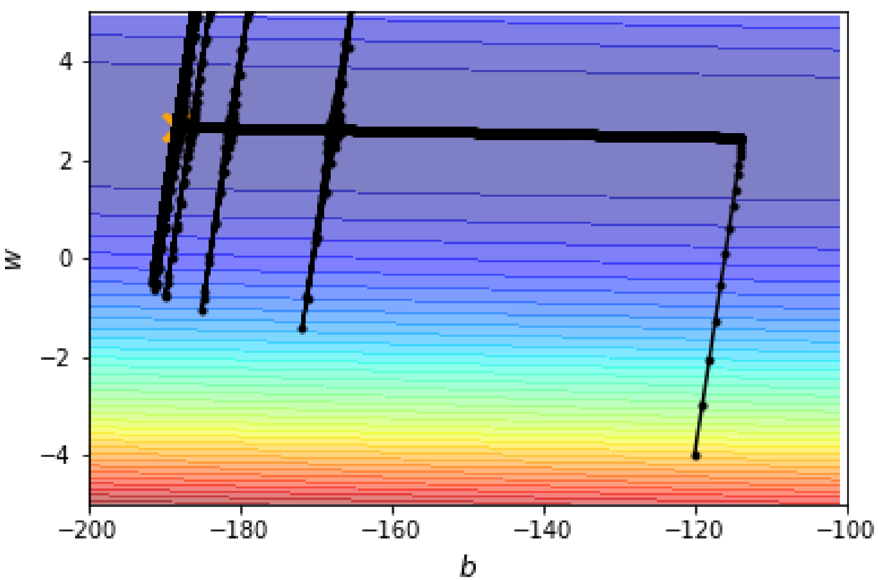
Learning Rate Scheduling

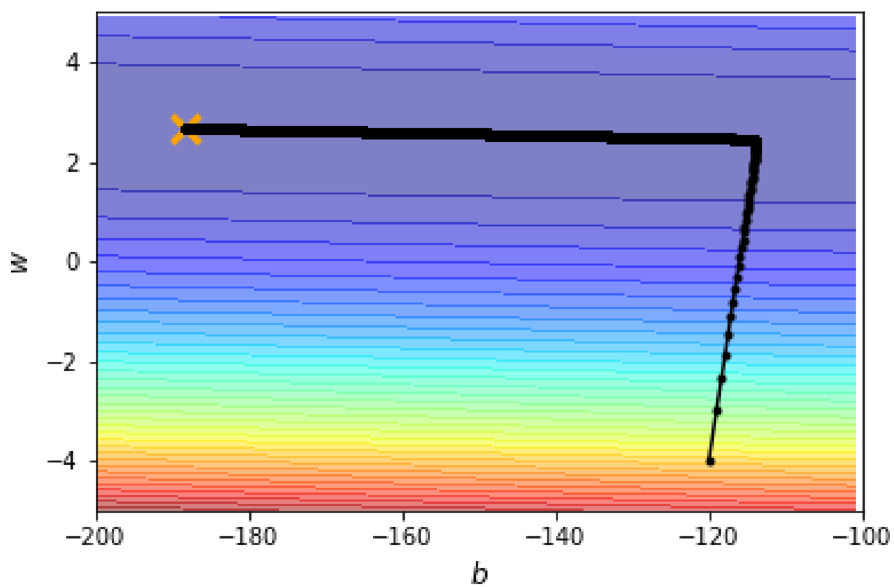
$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \frac{\boxed{\eta^t}}{\sigma_i^t} \boldsymbol{g}_i^t$$



Learning Rate Decay

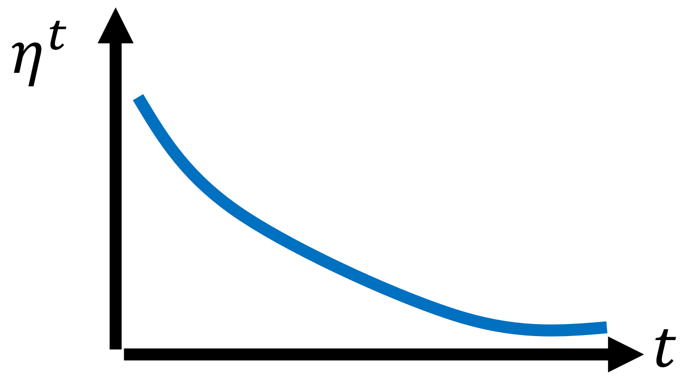
After the training goes, we are close to the destination, so we reduce the learning rate.





Learning Rate Scheduling

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \frac{\boxed{\eta^t}}{\sigma_i^t} \boldsymbol{g}_i^t$$



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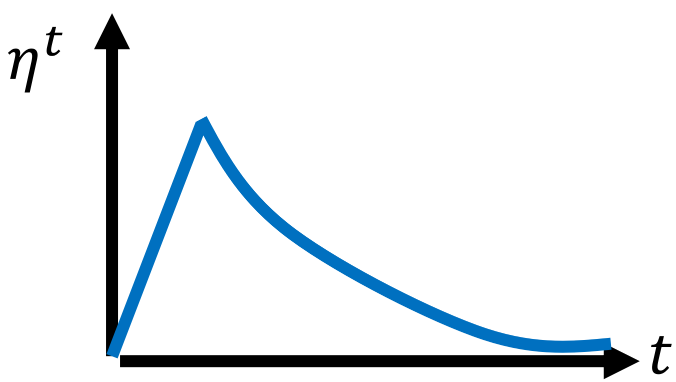


Warm Up

Increase and then decrease?

At the beginning, the estimate of σ_i^t has large variance.

We further explore $n = 18$ that leads to a 110-layer ResNet. In this case, we find that the initial learning rate of 0.1 is slightly too large to start converging[5]. So we use 0.01 to warm up the training until the training error is below 80% (about 400 iterations), and then go back to 0.1 and continue training. The rest of the learning schedule is as done previously. This 110-layer network converges well (Fig. 6, middle). It has *fewer* parameters than other deep and thin

[5]With an initial learning rate of 0.1, it starts converging (<90% error) after several epochs, but still reaches similar accuracy.

Residual Network

https://arxiv.org/abs/1512.03385

5.3 Optimizer

We used the Adam optimizer [17] with $\beta_1 = 0.9$, $\beta_2 = 0.98$ and $\epsilon = 10^{-9}$. We varied the learning rate over the course of training, according to the formula:

$$lrate = d_{\text{model}}^{-0.5} \cdot \min(step\_num^{-0.5}, step\_num \cdot warmup\_steps^{-1.5}) \tag{3}$$

This corresponds to increasing the learning rate linearly for the first $warmup\_steps$ training steps, and decreasing it thereafter proportionally to the inverse square root of the step number. We used $warmup\_steps = 4000$.

Transformer https://arxiv.org/abs/1706.03762

Please refer to **RAdam**

https://arxiv.org/abs/1908.03265

Summary of Optimization

(Vanilla) Gradient Descent

$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \eta \boldsymbol{g}_i^{\boldsymbol{t}}$$

Various Improvements

Learning rate scheduling

$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \frac{\eta^t}{\sigma_i^t} \boldsymbol{m}_i^{\boldsymbol{t}}$$

Momentum: weighted sum of the previous gradients

root mean square of the gradients

To Learn More ……

https://youtu.be/4pUmZ8hXlHM

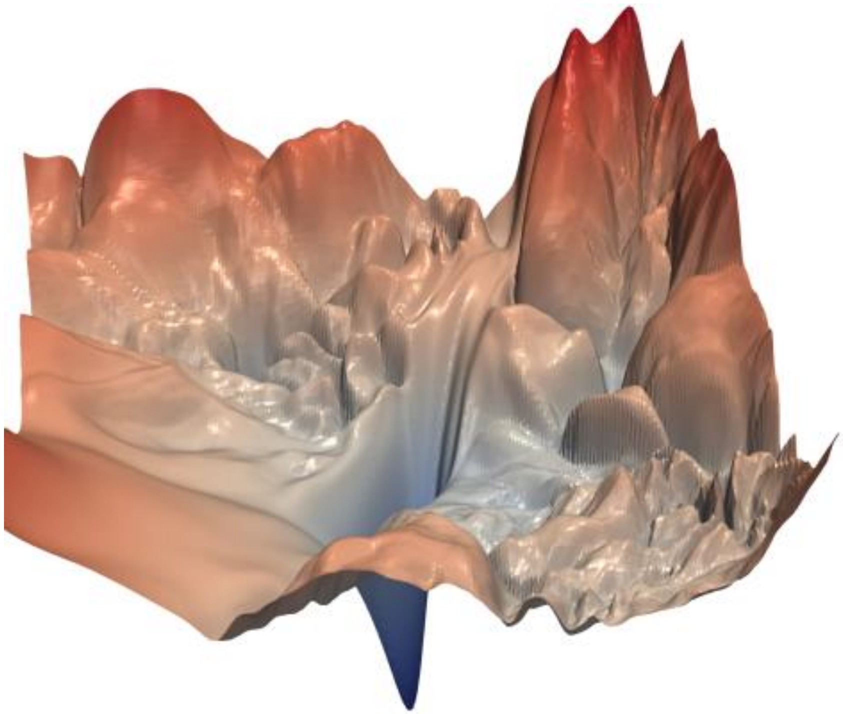
(in Mandarin)

https://youtu.be/e03YKGHXnL8

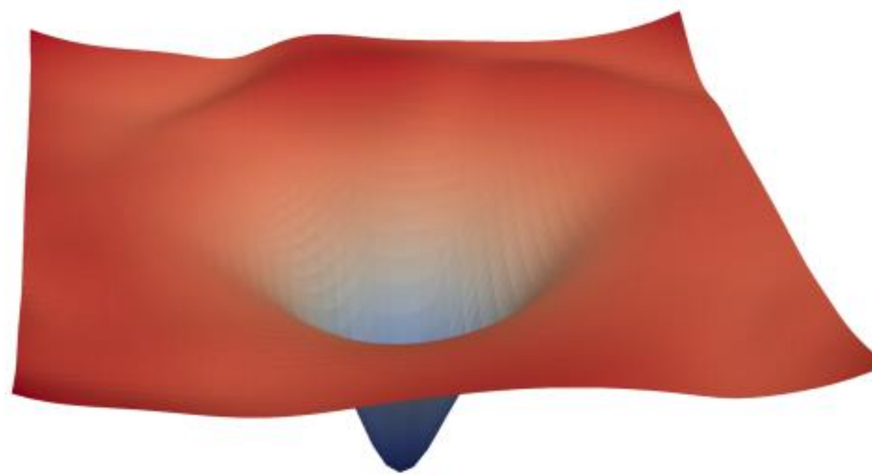
(in Mandarin)

Next Time

Source of image: https://arxiv.org/abs/1712.09913



Next time



Better optimization strategies:
If the mountain won't move,
build a road around it.

Can we change the error
surface?
Directly move the mountain!